Econometrics

Lecture 2

Nathaniel Higgins

ERS and JHU

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Plan for the lecture

- Administrative details
- Homework solutions
- Probability basics (review)
- Linear regression modeling with a single variable

Administrative stuff

- We have a TA!
- Introducing . . .

Administrative stuff

- We have a TA!
- Introducing . . . Fatima Alam
- Fatima will hold a weekly TA session in the computer lab (BOB 750) from 10:30a to 11:30a
- fatmeh@gmail.com
- The lab is reserved from 10:15a to 12:15p for the use of this class

Homework

- B.1 - B.5 (except part iii of B.3), B.7 - B.8, B.10, and C.1

- Q: Explain why the score of a person planning to take the SAT exam is properly viewed as a random variable

- Q: Explain why the score of a person planning to take the SAT exam is properly viewed as a random variable
- Because it is not deterministic! (not very satisfying)
- Because if you knew everything that you think could possibly impact your SAT score, you still wouldn't be totally sure what your SAT score would be without taking the test
- The SAT score is determined by "an experiment" (just aping the book)
- You cannot possibly know the SAT score until the exam is taken (the exam is the "experiment")

Homework
B.2

- $X \sim N(5, 4)$ Find the probabilities of:
- (i) $P(X \leq 6)$

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$$= 0.6915$$

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- (ii) continued ...

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- (ii) continued ...

$$1 - P\left(\frac{X-5}{2} < \frac{4-5}{2}\right) = 1 - P\left(Z < -\frac{1}{2}\right)$$

$$1 - P\left(Z < -\frac{1}{2}\right) = 1 - \Phi\left(-\frac{1}{2}\right)$$

$$1 - \Phi\left(-\frac{1}{2}\right) = 1 - 0.1151 = 0.8849$$

- (iii) $P(|X-5|>1)$

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$$= P(Z \geq \frac{1}{2}) + P(Z \leq -\frac{1}{2})$$

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$$= 1 - \Phi(\frac{1}{2}) + \Phi(-\frac{1}{2})$$

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$$=P(Z\geq \frac{1}{2})+P(Z\leq -\frac{1}{2})$$

$$=1-P(Z\leq \frac{1}{2})+P(Z\leq -\frac{1}{2})$$

$$=1-\Phi(\frac{1}{2})+\Phi(-\frac{1}{2})$$

$$=1-0.6915+0.3085=0.617$$

- 4,170 mutual funds over 10 years
- each fund has a 50/50 shot of beating S&P 500 each year
- each year's performance is independent of each other year
- Q: (i) P any particular fund outperforms index in all 10 years

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- Q: (i) P any particular fund outperforms index in all 10 years
- (0.5) in 1 year
- $(0.5) \times (0.5)$ in 2 years
- $(0.5) \times (0.5) \times \ldots \times (0.5)$ in 10 years
- $(0.5)^{10} = 0.00097$

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- P that at least one fund outperforms the market in all 10 years is the *complement* of the outcome that *no* one fund outperforms the market in all 10 years:

$$1 - P(\text{no fund outperforms the market})$$

- Use the binomial distribution to find the probability that there are no successes (no funds that beat the market in all 10 years) in 4,170 “tries” (4,170 firms that try to beat the market in all 10 years)

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$$f(4170, 0) = \binom{4,170}{0} \times (0.5^{10})^0 \times (1 - 0.5^{10})^{4,170}$$

Homework
B.3

- Use the binomial distribution to find the probability that there are no successes (no funds that beat the market in all 10 years) in 4,170 “tries” (4,170 firms that try to beat the market in all 10 years)

$$f(4170, 0) = \binom{4,170}{0} \times (0.5^{10})^{0} \times (1 - 0.5^{10})^{4,170}$$

- We want the *opposite* of no fund beats the market (which is that at least one fund beats the market), so we need to subtract this amount from 1 to get our answer

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$$1 - \left[\binom{4,170}{0} \times (0.5^{10})^0 \times (1 - 0.5^{10})^{4,170} \right]$$

Homework
B.3

- Use the binomial distribution to find the probability that there are no successes (no funds that beat the market in all 10 years) in 4,170 "tries" (4,170 firms that try to beat the market in all 10 years)

$$f(4170, 0) = \binom{4,170}{0} \times (0.5^{10})^{0} \times (1 - 0.5^{10})^{4,170}$$

- We want the *opposite* of no fund beats the market (which is that at least one fund beats the market), so we need to subtract this amount from 1 to get our answer

$$1 - \left[\binom{4,170}{0} \times (0.5^{10})^{0} \times (1 - 0.5^{10})^{4,170}\right]$$

$$= 0.9830$$

- $X =$ proportion of adults over 65 who are employed
- X has a cdf $F(x) = 3x^2 - 2x^3$
  - That is, $P(X \leq y) = 3y^2 - 2y^3$
- Q: What is the P that the employment rate is at least 0.6?

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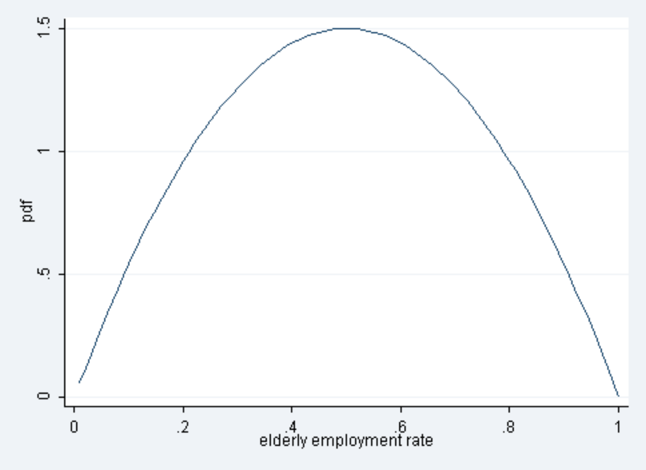
$$= 1 - (3 \times 0.6^2 - 2 \times 0.6^3) = 0.352$$

Homework
B.4



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Homework
B.5

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- Binomial(12,0.20)
- $P(X = 1) + P(X = 2) + \ldots P(X = 12)$

- Proportion believing innocent in the population is 0.2
- (i) P at least 1 juror of 12 thought Simpson innocent
- Binomial(12,0.20)
- $P(X = 1) + P(X = 2) + \ldots P(X = 12)$

$$f(x) = \binom{12}{x} \times 0.2^{x} \times 0.8^{11}$$

$$f(1) + f(2) + \ldots + f(12) =$$

- Proportion believing innocent in the population is 0.2
- (i) P at least 1 juror of 12 thought Simpson innocent
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$$f(x) = \binom{12}{x} \times 0.2^{x} \times 0.8^{11}$$

$$f(1) + f(2) + \ldots + f(12) =$$

- You're gonna drive yourself nuts doing this. Try the easy way

$$1 - f(0) = \binom{12}{0} \times (0.2^{0}) \times (0.8^{12}) = 0.93$$

- (ii) P at least 2 jurors of 12 thought Simpson innocent

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$$P(X \geq 2) = 1 - (P(X = 0) + P(X = 1))$$

- (ii) P at least 2 jurors of 12 thought Simpson innocent

$$P(X \geq 2) = 1 - (P(X = 0) + P(X = 1))$$

$$= 1 - (0.06871948 + 0.2061584) = 0.7251221$$

- 8 attempts, 0.74 chance of making each one
- $0.8 \times 0.74 = 5.92$
- Expected value calculation (long way)

- 0.74 chance of success. How many successes in 8 trials?
- Add up the probability of zero successes, multiplied by zero, plus the probability of one success, multiplied by one, and so on

$$\text{Successes in 8 trials} = \sum_{i=0}^{8} P(X = i) \times i$$

Homework
B.7

- $P(0 \text{ successes}) = f(0) = \binom{8}{0} + (0.74)^0 + (1 - 0.74)^8$

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Homework
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- $P(7 \text{ successes}) = f(7) = \binom{8}{7} + (0.74)^7 + (0.26)^1$
- $P(8 \text{ successes}) = f(8) = \binom{8}{8} + (0.74)^8 + (0.26)^0$

- Add these up to see the answer
- See spreadsheet B-7.xlsx for the answer

Homework
B.8

- 3 courses:
  - 3.5 gpa in a 2-credit course
  - 3.0 gpa in a 3-credit course
  - 3.0 gpa in a 4-credit course
- Before you start computing numbers, use what you know intuitively to give yourself a boost
- The overall gpa will be between which two numbers?

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- The overall gpa will be between which two numbers? 3.0 and 3.5
- Closer to which number?

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- Closer to which number? 3.0

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  - 3.0 gpa in a 4-credit course
- Before you start computing numbers, use what you know intuitively to give yourself a boost
- The overall gpa will be between which two numbers? 3.0 and 3.5
- Closer to which number? 3.0
- Which courses count more?

Homework
B.8

- 3 courses:
  - 3.5 gpa in a 2-credit course
  - 3.0 gpa in a 3-credit course
  - 3.0 gpa in a 4-credit course
- Before you start computing numbers, use what you know intuitively to give yourself a boost
- The overall gpa will be between which two numbers? 3.0 and 3.5
- Closer to which number? 3.0
- Which courses count more? I'm not answering this one for you. I refuse.
- How much more should the 3-credit course count than the 2-credit course?

Homework
B.8

- 3 courses:
  - 3.5 gpa in a 2-credit course
  - 3.0 gpa in a 3-credit course
  - 3.0 gpa in a 4-credit course
- Before you start computing numbers, use what you know intuitively to give yourself a boost
- The overall gpa will be between which two numbers? 3.0 and 3.5
- Closer to which number? 3.0
- Which courses count more? I'm not answering this one for you. I refuse.
- How much more should the 3-credit course count than the 2-credit course?
- $\frac{2\times 3.5+3\times 3.0|4\times 3.0}{9}=3.11$

Homework
B.10

- $E(GPA|SAT) = 0.70 + 0.002 \times SAT$

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- Q: What is the unconditional expectation of GPA?

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- Q: What is the unconditional expectation of GPA?
- A: Trick question. $E(GPA) = \textit{something}$
- That something is a constant number (like 0.70) but it's not 0.70

Homework
B.10

- $E(GPA|SAT) = 0.70 + 0.002 \times SAT$
- Q: What is the unconditional expectation of GPA?
- A: Trick question. $E(GPA) = something$
- That something is a constant number (like 0.70) but it's not 0.70
- (i) Expected GPA when SAT = 800; Find $E(GPA|SAT = 1,400)$

- $E(GPA|SAT) = 0.70 + 0.002 \times SAT$
- Q: What is the unconditional expectation of GPA?
- A: Trick question. $E(GPA) = \textit{something}$
- That something is a constant number (like 0.70) but it's not 0.70
- (i) Expected GPA when SAT = 800; Find $E(GPA|SAT = 1,400)$
- $E(GPA|SAT = 800) = 0.70 + 0.002 \times 800 = 2.3$

Homework
B.10

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- Q: What is the unconditional expectation of GPA?
- A: Trick question. $E(GPA) = \textit{something}$
- That something is a constant number (like 0.70) but it's not 0.70
- (i) Expected GPA when SAT = 800; Find $E(GPA|SAT = 1,400)$
- $E(GPA|SAT = 800) = 0.70 + 0.002 \times 800 = 2.3$
- $E(GPA|SAT = 1,400) = 0.70 + 0.002 \times 1,400 = 3.5$

Homework
B.10

- (ii) If avg. SAT is 1,100, what is avg. GPA?

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Homework
B.10

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Homework
B.10

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Homework
B.10

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- $E[0.70 + 0.002 \times SAT] = E(GPA)$
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Homework
B.10

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- $0.70 + 0.002 \times 1,100 = E(GPA)$
- $E(GPA) = 2.9$

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- $\frac{1}{4}[\mu + \mu + \mu + \mu] = \frac{1}{4}4\mu = \mu$

Homework
C.1

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Homework
C.1

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- $= \frac{1}{4^2} \sum_i \sigma^2$
- $= \frac{1}{4^2} 4\sigma^2 = \frac{1}{4}\sigma^2$

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- $\frac{1}{8}E(Y_1) + \frac{1}{8}E(Y_2) + \frac{1}{4}E(Y_3) + \frac{1}{2}E(Y_4)$
- $\frac{\mu+\mu+2\mu+4\mu}{8} = \frac{8\mu}{8} = \mu$

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- $\frac{\sigma^2(1+1+4+16)}{64} = \frac{22\sigma^2}{64} = 0.34\sigma^2$
- Compare to $0.25\sigma^2$ from the unweighted average

Homework

- Read chapters 1 and 2
- Start on problems at the end of chapter 2
- You will have to do 2.1 - 2.5 for the next problem set (due two weeks from today), so try to do those as you read